A fixed time-stepping scheme for rigid multi-body dynamics

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Nonsmooth multi-rigid-body dynamics

Nonsmooth rigid multibody dynamics (NRMD) methods attempt to predict the position and velocity evolution of a group of rigid particles subject to certain constraints and forces.

- non-interpenetration ($\Phi^{(j)}(q) \geq 0$).
- collision.
- joint constraints ($\Theta^{(i)}(q) = 0$).
- adhesion
- Dry friction – Coulomb model.
- global forces: electrostatic, gravitational.

The problems with event-based simulation

The simulation contains several types of events,

- Collisions.
- Contact take-off.
- Stick-slip transitions.

Event-detection simulators need to stop and interpret each event. A large number of collisions may result in the simulation becoming extremely slow.

Fixed, large, time-step simulation?

Fixed timestep simulation: The time-step does not go to 0 because of either events or stability reasons. This can be achieved one of the following ways:

- Use a smoothed approach for contact/collision/friction and integrate explicitly. The timestep may get prohibitively small for stability reasons.
- Use a smoothed approach plus an implicit method.
- Use a hard constraint, complementarity approach, without collision detection or backtracking and with special restitution rule. **This work.**

We show that the last two approaches are equivalent in the limit! (see also Kumar, Song and Pang 2003). So we concentrate on the second approach.

Linearization method I

Consider penalty method for one contact constraint, $\Phi^{(1)}(q) \geq 0$.

$$\Phi^{(1)}(q) \geq 0 \;\text{ enforced by penalty force }\; \theta^{(1)}(q) = \gamma^{(1)} \left(\Phi_{-}^{(j)}(q) \right)^2$$

where $\gamma^{(1)}$ is a very large parameter.

Dynamics (for the frictionless case becomes)

$$\begin{aligned} \frac{dq}{dt} &= v. \\ M\frac{dv}{dt} &= k(t,q,v) + \theta^{(1)}(q)\nabla_q \Phi^{(1)}(q). \end{aligned}$$

Apply backward Euler, where $\Phi^{(1)}(q^{(l+1)})$ is replaced by its linearization

$$\Phi^{(1)}(q^{(l+1)}) \approx \Phi^{(1)}(q^{(l)}) + h_l \nabla_q \Phi^{(1)}(q^{(l)})^T v^{(l+1)}$$

Take the limit as time step h_l is fixed and $\gamma^{(1)} \to \infty$ and ....

Linearization method II

... we obtain a complementarity model!:

$$
\begin{aligned}
q^{(l+1)} &= q^{(l)} + h_l v^{(l+1)}. \\
M\frac{v^{(l+1)} - v^{(l)}}{h_l} &= k(t^{(l)}, q^{(l)}, v^{(l)}) + c^{(1),(l+1)} \nabla_q \Phi^{(1)}(q^{(l+1)}) \\
0 \leq c^{(1),(l+1)} &\perp \Phi^{(1)}(q^{(l)}) + h_l \nabla \Phi^{(1)}(q^{(l)})^T v^{(l+1)} \geq 0
\end{aligned}
$$

- Solution to the "do not backtrack at collision" and constraint stabilization problem: Use the complementarity model with replacements

  $$\Phi^{(j)}(q^{(l)})^T(q) \geq 0 \Longrightarrow \Phi^{(j)}(q^{(l)}) + \gamma h_l \nabla \Phi^{(j)}(q^{(l)})^T v^{(l+1)} \geq 0.$$

  $$\Theta^{(i)}(q^{(l)})^T(q) = 0 \Longrightarrow \Theta^{(i)}(q^{(l)}) + \gamma h_l \nabla \Theta^{(i)}(q^{(l)})^T v^{(l+1)} = 0.$$

  Here $\gamma \in (0, 1]$. $\gamma = 1$ corresponds to exact linearization.

- **This also shows that the LCP method and the implicit approach of penalty method are equivalent!**

Treating Partially Elastic Collisions

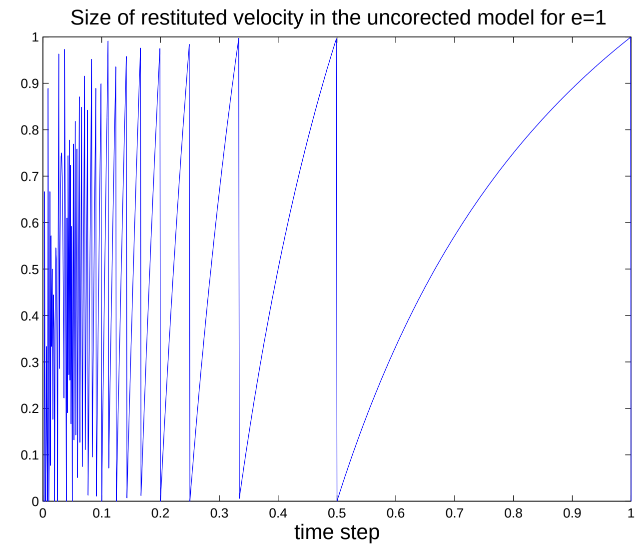
- It can be shown that this coresponds to 0 restitution coefficient. Good start, but extra modeling is necessary.
- A portion of the collision impulse (Poisson law) or normal velocity (Newton law) is restituted to the system.
- Newton's law is more efficient computationally because it can enforced in one subproblem (if collision has occured in the previous step):

$$\frac{\Phi^{(j)}(q^{(l)})}{h} + \nabla \Phi^{(j)^T}(q^{(l)}) v^{(l+1)} + e_N \nabla \Phi^{(j)^T}(q^{(l-1)}) v^{(l)} \geq 0.$$

- Used with collision detection in (Stewart and Trinkle 96), (Stewart 00) and without collision detection in (Moreau89), (Moreau and Jean94), (Jean99).

Irregular behavior of the model

- However, odd results appear if we apply it in connection with the linearization method without collision detection.
- Example: one particle colliding with a wall. The normal velocity, that is essential to the restitution model, is all over the place.



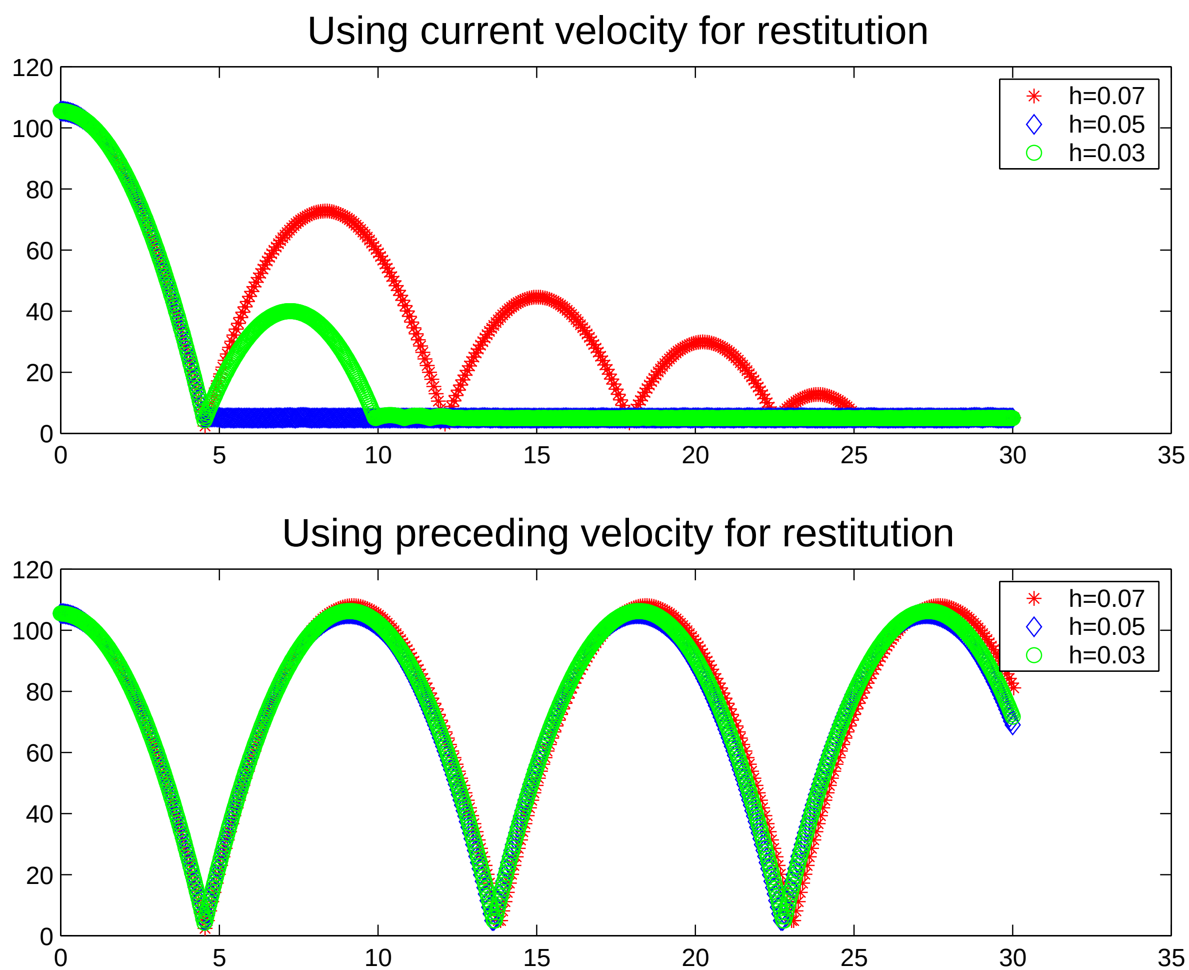
Getting to a workable restitution model

- A simple fix, which works in most cases, (though not provable) is to lag the normal velocity by 1.
- For the LCP with linearization, the change becomes

$$\frac{\Phi^{(j)}(q^{(l)})}{h} + \nabla\Phi^{(j)^T}(q^{(l)})v^{(l+1)} + e_N \nabla\Phi^{(j)^T}(q^{(l-2)})v^{(l-1)} \geq 0.$$

- For one isolated collision it can be shown that this rule captures the exact behavior in the limit as the time step goes to 0. But no theory exists in general for this case, though simulations behave according to expectations.

Comparison between the two approaches



Time-stepping scheme

Euler method, half-explicit in velocities, discretization of friction cone, Maximum dissipation principle enforced through optimality conditions.

$$M(v^{l+1} - v^{(l)}) - \sum_{i=1}^{m} \nu^{(i)} c_{\nu}^{(i)} - \sum_{j \in \mathcal{A}} (\quad n^{(j)} \quad c_n^{(j)} + D^{(j)} \beta^{(j)}) = hk$$

$$\nu^{(i)^T} v^{l+1} = -\gamma \frac{\Theta^{(i)}}{h}, \qquad i = 1, 2, \ldots, m$$

$$\rho^{(j)} = n^{(j)^T} v^{l+1} \geq -\gamma \frac{\Phi^{(j)}(q)}{h} + e^{(j)} v_N^{(j),l-1}, \quad \perp \quad c_n^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\sigma^{(j)} = \lambda^{(j)} e^{(j)} + D^{(j)T} v^{l+1} \geq 0, \quad \perp \quad \beta^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\zeta^{(j)} = \mu^{(j)} c_n^{(j)} - e^{(j)^T} \beta^{(j)} \geq 0, \quad \perp \quad \lambda^{(j)} \geq 0, \quad j \in \mathcal{A}.$$

Here $\nu^{(\mathbf{i})} = \nabla \boldsymbol{\Theta}^{(\mathbf{i})}$, $n^{(j)} = \nabla \Phi^{(j)}$. h is the time step. The set $\mathcal{A}$ consists of the active constraints. Modification of (Anitescu and Potra,1997) and (Stewart and Trinkle, 1996),

Choice of the active set

- We need a special definition of the active set to avoid collision detection.
- Definition: The active set $\mathcal{A} = \{j | 1 \leq j \leq n, \quad \Phi^{(j)}(q^{(l)}) \leq \hat{\epsilon}\}$.
- ϵ should be corelated with the largest expected value of the velocity.

Matrix Form of the Integration Step

$$\begin{bmatrix} M & -\tilde{\nu} & -\tilde{n} & -\tilde{D} & 0 \\ \tilde{\nu}^T & 0 & 0 & 0 & 0 \\ \tilde{n}^T & 0 & 0 & 0 & 0 \\ \tilde{D}^T & 0 & 0 & 0 & \tilde{E} \\ 0 & 0 & \tilde{\mu} & -\tilde{E}^T & 0 \end{bmatrix} \begin{bmatrix} v^{(l+1)} \\ \tilde{c}_\nu \\ \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} -Mv^{(l)} - hk \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix}^T \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} = 0, \qquad \begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} \geq 0.$$

Note Replacing 0 by $-\tilde{\mu}$ makes the problem PSD (a QP)!

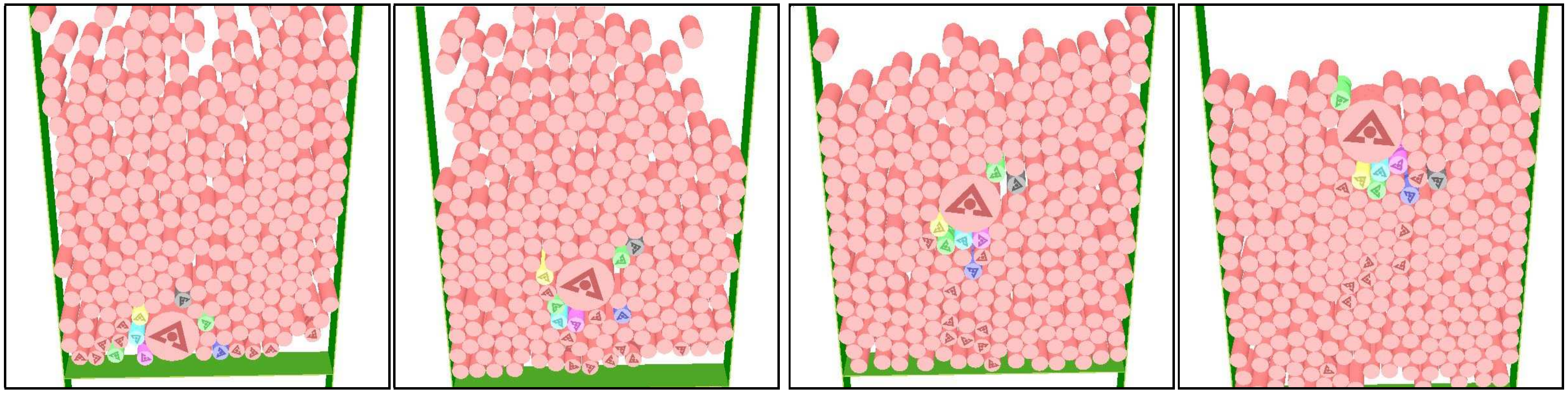
About the convex relaxation

- It is shown that, as $h \to 0$ it converges to a weak solution of the dynamics equations (Anitescu 03).
- It is shown that constraint stabilization is achieved for the original and relaxed scheme (Anitescu and Hart, 03), (Anitescu, Miller and Hart 03).
- At least when the restitution coefficient is 0, the energy does provably not increase.
- However, experimentally, even with general restitution coefficients the energy does not increase (results included in the proceedings paper).
- Recent advances in interior-point methods, where conic constraints are treated directly, may be used to avoid discretization of the friction cone.

Granular matter

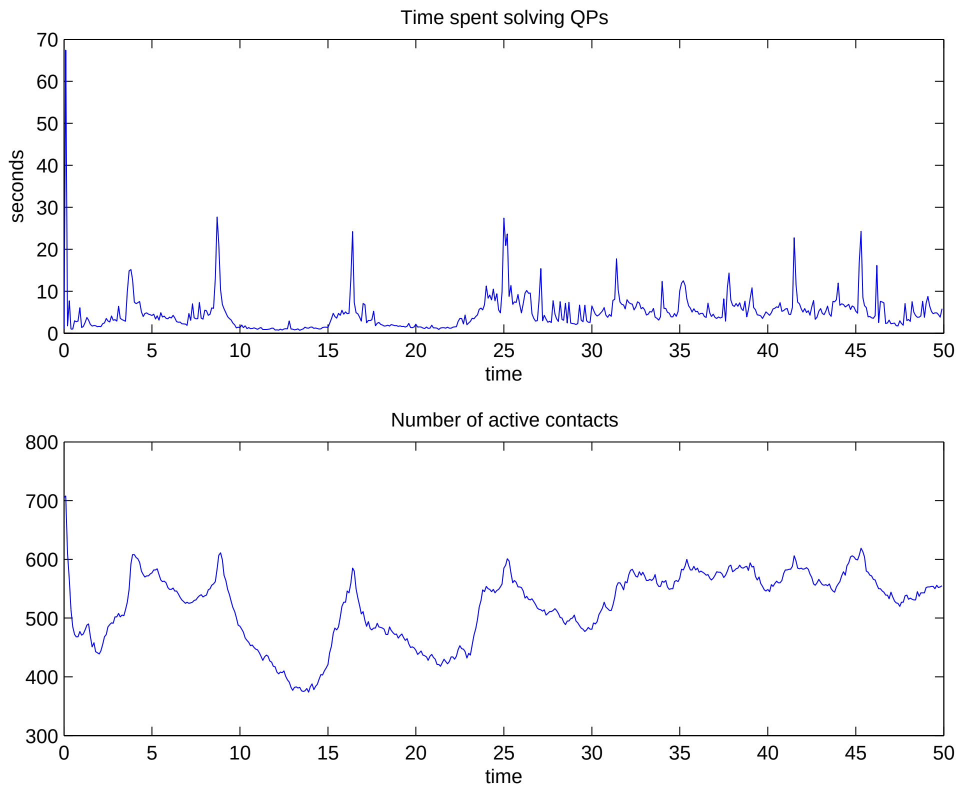
- Sand, Powders, Rocks, Pills are examples of granular matter.
- The range of phenomena exhibited by granular matter is tremendous. Size-based segregation, jamming in grain hoppers, but also flow-like behavior.
- There is still no accepted continuum model of granular matter.
- Direct simulation methods (discrete element method) are still the most general analysis tool, but they are also computationally costly.
- The favored approach: the penalty method which works with time-steps of microseconds for moderate size configurations.

Brazil nut effect simulation



- Time step of 100ms, for 50s. 270 bodies.
- Convex Relaxation Method. One QP/step. No collision backtrack.
- Friction is 0.5, restitution coefficient is 0.5.
- Large ball emerges after about 40 shakes. Results in the same order of magnitude as MD simulations (but with 4 orders of magnitude larger time step).

Brazil nut effect simulations performance



Conclusions and future work

- We construct a fixed time step scheme that can acommodate contact, partially elastic collision, and friction.
- The modeling difficulties at collision are resolved by lagging the data.
- An assymptotically consistent convex relaxation approach is available, and it may be used without discretization of the friction cone.
- In the near future, we plan to modify the scheme to deal with nonsmooth shapes.